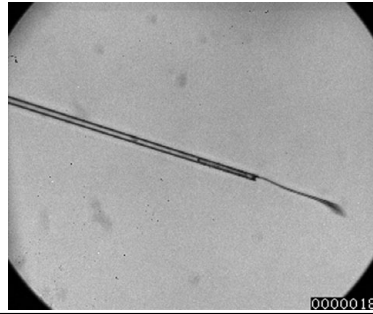# Cavitation in linear bubbles

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Recent work has developed a beautiful model system for studying the energy focusing and heating power of collapsing bubbles. The bubble is effectively one-dimensional and the collapse and heating can be quantitatively measured. Thermal effects are shown to play an essential role in the time-dependent dynamics.

Keywords. Bubble dynamics, Cavitation

1. Introduction

In recent years, there has been enormous excitement and speculation about the ability of bubbles to focus energy. The discovery of single bubble sonoluminescence, in which a single acoustically forced bubble collapses and produces visible light, led to a flurry of work aimed at understanding the limits of the energy focusing power of small bubbles (Moss *et al.* 1996; Barber *et al.* 1997; Brenner, Hilgenfeldt & Lohse 2002). Initial estimates of the temperature inside a collapsed bubble were tempered by the realization of the limits imposed by viscous and thermal effects, as well as the gas and water vapour content of the bubble. Current estimates of the maximum temperature inside collapsed bubbles are of the order of 15 000K (Gompf *et al.* 1997; Flannigan & Suslilck 2005).

These estimates are consistent with experimental measurements of the properties of light emission characteristics (for a review see Brenner *et al.* 2002). However, it has thus far proven to be quite impossible to compare theoretical predictions of a collapsing bubble directly with experiments. The sizes of the bubbles are simply too small, and the collapse rates are too rapid. Adding to this difficulty is the fact that the thermal heating of the gas inside a collapsing bubble has very little impact on the bubble dynamics itself, so measurements of the bubble dynamics alone do not probe thermal effects.

The initial studies of sonoluminescence used bubbles injected into a continuous acoustic field. Later on, an alternative method for creating transient bubbles was used, the focusing of a laser spot in the liquid (Brewer & Rieckhoff 1964; Lauterborn & Bolle 1975; Vogel, Lauterborn & Timm 1989; Ohl, Lindau & Lauterborn 1998; Baghdassarian, Tabbert & Williams 1999). The energy deposited by the laser vaporizes a small amount of liquid generating a rapidly expanding vapour bubble, whose ultimate collapse creates a pulse of light. From both the spectrum and spectral lines, the temperature of the gas (vapour) in the bubble has been estimated at about 7800K (Baghdassarian *et al.* 2001). Recent work of Zwaan *et al.* (2007) used laser-induced bubbles to produce cavitation in a microfluidic device. Quantitative
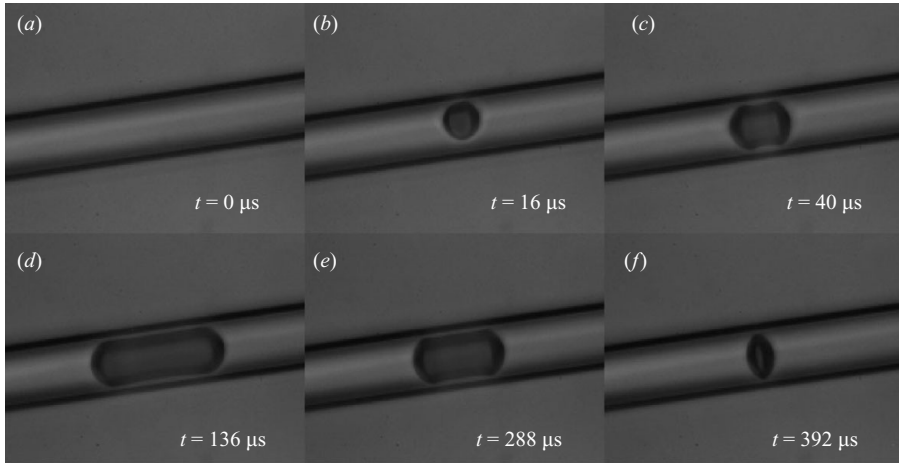
FIGURE 1. Representative frames showing the growth and collapse of a vapour bubble inside a microtube with inner diameter 50 μm and length 27 mm; the absorbed laser energy is 11.6 μJ.

studies of vapour bubbles date to the pioneering work of Plesset & Zwick (1954), which were experimentally tested by Dergarabedian (1960).

Sun *et al.* (2009, this issue, Vol. 632, pp. 5–16) create bubbles that expand and collapse along a linear microfluidic tube (Yuan, Oguz & Prosperetti 1999; Ajaev, Homsy & Morris 2002). Strikingly, by measuring the length $X(t)$ of the bubble, Sun *et al.* (2009) demonstrate that the dynamics of the bubble depend critically on the thermal evolution of the gas in the bubble, and the surrounding liquid, due to the diffusion and dissipation of the heat input by the laser.

2. Overview

A laser pulse is focused at the centre of a microtube to create a 'linear' bubble (figure 1). The bubble initially grows spherically to fill the radius of the tube (figure 1*a*, *b*), then expands along the axis of the tube (figure 1*c*). A thin liquid layer remains around the boundaries of the bubble, whose expansion is essentially one-dimensional. The length of the bubble $X(t)$ initially grows, then collapses to a point (figure 1*e*, *f*). Figure 2 shows the variation of $X(t)$ in a tube with diameter $D = 50\,\mu\text{m}$.

The length $X(t)$ of the bubble obeys

$$\ell_L \rho_L \frac{\text{d}^2 X}{\text{d}t^2} + \mathscr{R}\frac{\text{d}X}{\text{d}t} = p_v(t) - P_\infty. \tag{2.1}$$

Here ℓ_L is the length of the liquid column, ρ_L is the density of the liquid, and $\mathscr{R}$ is the drag coefficient. Sun *et al.* (2009) approximate $\mathscr{R} = 32\mu\ell_L/D^2$, where μ is the liquid viscosity. The acceleration of the liquid in the channel (first term) is balanced by the damping of the bubble due to viscous forces (second term) and the pressure force on the bubble, due to the difference between the vapour pressure p_v inside the bubble and the pressure far away P_∞.

Sun *et al.* (2009) consider two different models for $p_v(t)$. In the first, they assume that the vapour pressure experiences an initial spike due to the laser. The model contains two free parameters, the maximum pressure as well as the time at which the pressure is
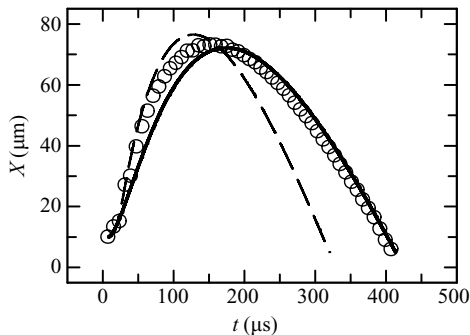
FIGURE 2. Comparison of the bubble length $X(t)$ versus time as measured and predicted by the models for a tube with inner diameter 50 μm. The open symbols are the experimental results; the dashed line plots the predictions of a step-function pressure model, and the solid line plots the predictions of a model with time-dependent thermal effects.

high. Figure 2 shows a comparison of this model with the experiments (dashed line). To understand the difference between this model and the experiment of Sun *et al.* (2009) rewrite (2.1) and solve for $p_v(t)$. Using their experimental measurements for $X(t)$ they can therefore effectively measure the time dependence of $p_v(t)$, which depends on the temperature inside the bubble, in a fashion that is directly known from equilibrium thermodynamics. Therefore they can estimate the quantitative disagreement.

The temperature in the bubble appears not to asymptote to the initial value of 25°C, but instead ends up near 60°C. This suggests that thermal effects are important. Sun *et al.* (2009) assume that the laser spot deposits a localized temperature distribution, and then couple the bubble dynamics to an evolving temperature field. This model produces much better agreement with experiments (figure 2, solid line), including accurate prediction of the temperature in the bubble.

3. Future

In contrast to the spherically symmetric examples described in §1, the linear bubble is strongly influenced by the deposited temperature field and the heat transfer. By changing the diameter of the tube, and the input laser energies, the bubble velocity can be changed from $0.1\,\mathrm{m\,s^{-1}}$ to upwards of $1\,\mathrm{m\,s^{-1}}$ – a very high velocity for flow in a 24.9 μm tube! However, these collapsing bubbles have internal temperatures upwards of 170°C, much smaller than those in the spherical case. Without spherical focusing, the energy focusing power of the bubble is likely limited; but the collapse, and associated temperature increase of a bubble might be greatly enhanced if it was surrounded on both sides by bubbles in their expanding phase.

As previously noted, laser-induced linear bubbles might also find uses as little microfluidic pumps (Yin & Prosperetti 2005*a*, *b*). The image beside the title shows a high-speed jet with maximum velocity $100\,\mathrm{m\,s^{-1}}$, produced by the growth of a vapour bubble near the end of a microtube with a diameter of 6 μm. The vapour bubble was created at 250 μm from the end of the tube by a focused laser pulse. The energy absorbed by the working fluid in the tube is 5 μJ. Such extremely rapid velocities created by a bubble in the liquid will propagate throughout the microfluidic device, and might be used as a basis for forcing or mixing small-scale flows.

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